Journal of Engineering Physics and Thermophysics, Vol. 68, No. 2, 1995

MATHEMATICAL SIMULATION OF HEAT CONDUCTION PROCESSES IN AN ABRASIVE TOOL IN THE PRESENCE OF PHYSICOCHEMICAL TRANSFORMATIONS IN IT

A. K. Tsokur, A. Ya. Tsokur, and
V. G. Gavrilov

UDC 536.2:621.923

Based on the heat conduction equation, mathematical models (one- and two-dimensional) have been developed to describe a nonstationary temperature field in an abrasive tool. The physicochemical transformations occurring in it and of the mobility of its peripheral surface are taken into account.

An important factor for the efficiency of the process of grinding, the quality of machining, and the durability of an abrasive tool, itself a multicomponent system, is the strength of the bridges of the bond and its thermal state. It is thought that the thermophysical characteristics of the material of the disk and the kinematics of the process influence the quantity of heat that gains access to the disk [1, 2]. Recently the possibility of the occurrence of chemical reactions in the zone of grinding has been established [3]. The thermal effects of these reactions, just as those of physical processes, are capable of exerting a noticeable effect on the formation of the temperature fields.

In [2] the problem of finding the temperature field in an abrasive disk during cyclic heating of it was solved. But such important factors as the geometric dimensions of the disk and the physicochemical transformations in it were not taken into account there. In the present work we suggest two mathematical models of the process of heating an abrasive disk that generalize the model proposed in [2].

We consider a rapidly rotating abrasive disk (Fig. 1). A heat source acts operative on a small portion of its peripheral surface; beyond the zone of the influence of the source the surface of the disk is cooled by convective heat transfer, i.e., the disk is loaded by an asymmetric heat source. During heating, physicochemical processes take place in the interior of the instrument that are accompanied by the liberation or absorption of heat. Let us denote the enthalpy of the physicochemical processes by H_1 and the concentration of the reagent by v. Then the power of the distributed sources or sinks of heat at the time t will be $H_1 \partial v/\partial t$.

Since the disk rotates, all of the points of the peripheral surface pass successively through the stages of cooling and heating, i.e., all of these points are subjected to identical temperature conditions. Thus, the problem can be considered to be axisymmetric, with the graph of the time dependence of temperature for an arbitrary point of the periphery having the form shown in Fig. 2 (the curve $f_1(t)$). We consider, for example, a typical heating-cooling cycle. A small portion Γ_2^+ of the periphery is heated in the time $\Delta t = t_2 - t_1$ from the initial temperature T_1 to the final temperature T_2. This portion cools off in the time $\Delta t = t_3 - t_2$ to the final temperature $T_3 > T_1$, and so on in cycles. After a certain interval of time t_0 the process of heating of the peripheral surface reaches a steady state, i.e., after the time t_0 the temperature of the periphery is established at a fixed level T_{lev}.

We employ the following simplification: we replace the curve $f_1(t)$ by the smoothed curve $f_2(t)$. The smoothing can be done by the least-squares method by replacing $f_1(t)$ by a parabola of the form $at^2 + bt + c$. This replacement does not change the qualitative picture of the temperature field in the abrasive disk.

Thus, on the peripheral surface Γ_2 (see Fig. 1) we can write the following boundary condition:

$$T = f_2(t) \quad \text{on} \quad \Gamma_2, \tag{1}$$

where

V. Ya. Chubar' Zaporozhe Machine Construction Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 68, No. 2, pp. 306-311, March-April, 1995. Original article submitted February 18, 1992.

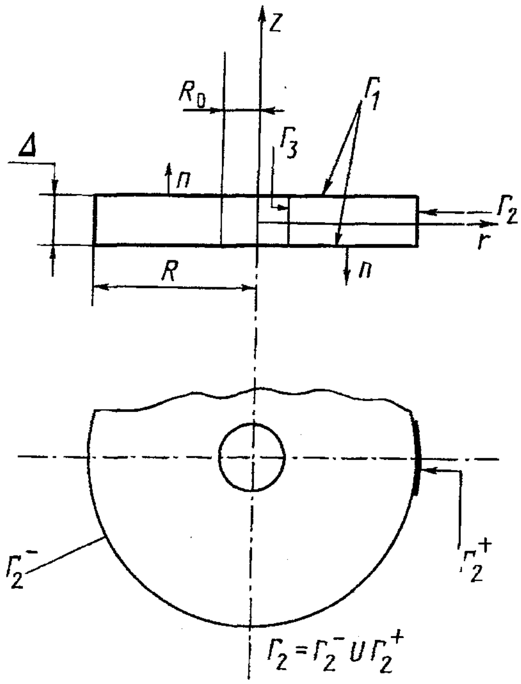

Fig. 1. Abrasive disk: n, vector of the unit external normal to the surface Γ_1; Γ_1, Γ_2, Γ_3, surfaces bounding the disk, with $\Gamma_2 = \Gamma_2^+ \cup \Gamma_2^-$, where Γ_2^+ is the heated portion of the working surface of the disk and Γ_2^- is the portion of the surface being cooled.

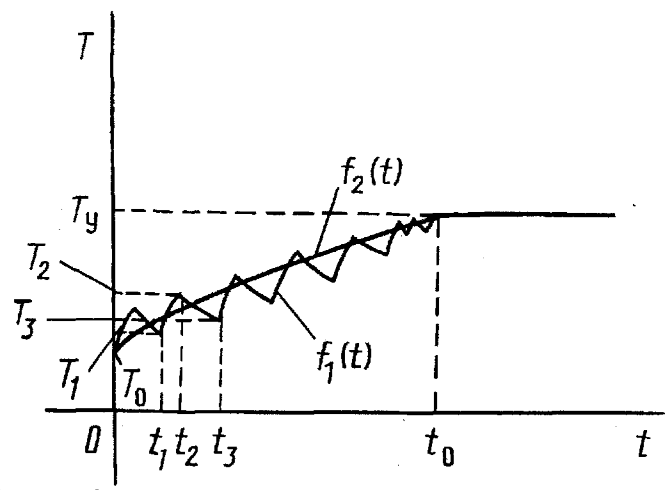

Fig. 2. Graphs of the functions $f_1(t)$ and $f_z(t)$ describing the change in the temperature of the peripheral (working) surface with time.

$$f_2(t) = \begin{cases} at^2 + bt + c, & 0 \le t \le t_0, \\ T_{\text{lev}}, & t \ge t_0. \end{cases}$$

On the side surfaces Γ_1 of the disk, heat exchange with the surrounding medium takes place, i.e.,

$$\lambda_1 \frac{\partial T}{\partial n} = -\alpha (T - T_\infty) \quad \text{on} \quad \Gamma_1. \tag{2}$$

The disk is clamped on a metal axis, and consequently, Γ_3 is the place of contact of the disk with the metal axis. Therefore, it is natural to assume that there is a distributed thermal capacity on Γ_3, i.e.,

$$\lambda_1 \frac{\partial T}{\partial \hat{r}} = R_0 c_2 \rho_2 \frac{\partial T}{\partial t} \quad \text{on} \quad \Gamma_3. \tag{3}$$

The initial condition is

$$T(\hat{r}, z, 0) = T_0 \quad \text{at} \quad R_0 \le \hat{r} \le R, \quad -\frac{\Delta}{2} \le z \le \frac{\Delta}{2}.$$

Supplementing boundary conditions (1)-(3) with the heat conduction equation written in cylindrical coordinates $\hat{r}$, φ, z, we arrive at the following mathematical model of the process of heating the abrasive disk:

$$c_1\rho_1 \frac{\partial T}{\partial t} = \frac{1}{\hat{r}} \frac{\partial}{\partial \hat{r}} \left(\lambda_1 \hat{r} \frac{\partial T}{\partial \hat{r}} \right) + \frac{\partial}{\partial z} \left(\lambda_1 \frac{\partial T}{\partial z} \right) + H_1 \frac{\partial v}{\partial t},$$

$$t > 0, \quad R_0 \le \hat{r} \le R, \quad -\frac{\Delta}{2} \le z \le \frac{\Delta}{2}, \tag{4.1}$$

$$T = f_2(t), \quad \hat{r} = R, \tag{4.2}$$

$$\lambda_1 \frac{\partial T}{\partial n} = -\alpha (T - T_\infty), \quad z = \pm \frac{\Delta}{2}, \tag{4.3}$$

$$\lambda_1 \frac{\partial T}{\partial \hat{r}} = R_0 c_2 \rho_2 \frac{\partial T}{\partial t}, \quad \hat{r} = R_0, \tag{4.4}$$

$$T = T_0 \quad \text{at} \quad t = 0, \quad R_0 \le \hat{r} \le R, \quad -\frac{\Delta}{2} \le z \le \frac{\Delta}{2}. \tag{4.5}$$

Making certain further assumptions, it is possible to reduce mathematical model (4.1)-(4.5) to a one-dimensional model in spatial variables.

Due to the low thermal conductivity of the abrasive disk, we will assume that the temperature of the disk does not change over its thickness, i.e., we will assume that the unknown temperature field T is a function only of the radial coordinate $\hat{r}$. Averaging the heat conduction equation along the axis $0z$ under these conditions and taking into account boundary condition (4.3), we obtain a modified heat conduction equation:

$$c_1\rho_1 \frac{\partial T}{\partial t} = \frac{1}{\hat{r}} \frac{\partial}{\partial \hat{r}} \left(\lambda_1 \hat{r} \frac{\partial T}{\partial \hat{r}} \right) - \frac{2\alpha}{\Delta} (T - T_\infty) + H_1 \frac{\partial v}{\partial t}. \tag{5}$$

Hereafter it is preferable to work with dimensionless quantities:

$$r = \hat{r}/R, \quad c = c_1/c_0, \quad \rho = \rho_1/\rho_0, \quad \lambda = \lambda_1/\lambda_0,$$

$$H = H_1/H_0, \quad u = (T - T_\infty)/(T_{\text{lev}} - T_\infty), \quad \tau = (\lambda_1/c_1\rho_1 R^2)\, t, \quad h = H_0/c_0\,(T_{\text{lev}} - T_\infty), \tag{6}$$

where c_0, ρ_0, λ_0, H_0 are the temperature-independent components of the quantities c_1, ρ_1, λ_1, H_1 (i.e., $c_1 = c_0 + \hat{c}(T)$, $\lambda_1 = \lambda_0 + \hat{\lambda}(T)$, and so on); u is the dimensionless temperature function; τ is the Fourier number.

Equation (5) in dimensionless form is

$$\left(c\rho - hH \frac{\partial v}{\partial u} \right) \frac{\partial u}{\partial \tau} = \lambda \frac{\partial^2 u}{\partial r^2} + \frac{\lambda}{r} \frac{\partial u}{\partial r} + \frac{\partial \lambda}{\partial u} \left(\frac{\partial u}{\partial r} \right)^2 - \frac{2\alpha R^2 u}{\lambda_0 \Delta}. \tag{7}$$

Supplementing Eq. (7) with boundary conditions (4.2) and (4.4) and initial condition (4.5) (assuming that $T = T_0$), we arrive at a one-dimensional and strongly nonlinear mathematical model of the process of heating the abrasive disk:

$$\left(c\rho - hH \frac{\partial v}{\partial u} \right) \frac{\partial u}{\partial \tau} = \lambda \frac{\partial^2 u}{\partial r^2} + \frac{\lambda}{r} \frac{\partial u}{\partial r} + \frac{\partial \lambda}{\partial u} \left(\frac{\partial u}{\partial r} \right)^2 - \frac{2\alpha R^2 u}{\lambda_0 \Delta}, \quad \tau > 0, \tag{8.1}$$

$$u = \frac{f_2(t) - T_\infty}{T_{\text{lev}} - T_\infty}, \quad r = 1, \quad (R_0/R) \le r \le 1, \tag{8.2}$$

$$\lambda \frac{\partial u}{\partial r} = (c_2\rho_2/c_0\rho_0) \frac{R_0}{R} \frac{\partial u}{\partial r}, \quad r = R_0/R, \tag{8.3}$$

$$u = 0 \quad \text{at} \quad \tau = 0, \quad (R_0/R) \le r \le 1. \tag{8.4}$$

Making several more simplifying assumptions, we may obtain additional information about the processes occurring in the disk. In fact, for large specific heat loads, high heating rates, and a low heat conduction coefficient (which occurs during abrasive treatment) it is natural in the first approximation to consider the region of physicochemical transformations to be a surface, i.e., an interface separating the substance that reacted from that which has not reacted. To derive the law of motion of this interface we shall avail ourselves of the solution of the Stefan problem [4]. We arbitrarily take the temperature distribution along the radius in the form:

$$T = T_{\text{lev}} - \frac{T_{\text{lev}} - T_r}{\hat{b}} \hat{r},$$

where $\hat{b}$ is the coordinate of the position of the moving interface of physicochemical transformations at a given instant; T_r is the temperature of the moving interface. Then the velocity of the interface in moving from the periphery to the center of the disk can be determined approximately, just as in [4]:

$$H_0 \rho_0 \frac{d\hat{b}}{dt} = - \frac{\lambda_0 (T_{\text{lev}} - T_r)}{\hat{b}}.$$

Using formulas (6), we make the above equation dimensionless:

$$\frac{db}{d\tau} = - \frac{u_r}{h} \frac{1}{b}, \tag{9}$$

where

$$b = \hat{b}/R, \quad u_r = \frac{T_{\text{lev}} - T_r}{T_{\text{lev}} - T_\infty}.$$

The initial conditions for integrating Eq. (9) are the following:

$$b = 1 \quad \text{at} \quad \tau = 0. \tag{10}$$

The integration of Eq. (9), which satisfies initial conditions (10), yields

$$b = \sqrt{\left(1 - \frac{2u_r}{h}\tau\right)},$$

i.e., at the beginning of the process (at small values of τ) the rate of decrease of the radius of the disk due to its wear is small. As the duration of the occurrence of the process (the time of disk service) increases, the rate of its wear increases steadily. The enthalpy of the physicochemical transformations exerts a similar influence: the more endothermic the transformations are, the smaller the rate of wear. This is confirmed by experimental data.

Taking into account the mobility of the front of the physicochemical processes and the cyclic character of the heating of the disk, we reduce Eq. (7) to fixed boundaries by means of a linear transformation:

$$r(\tau) = (R_0/R) + x\,[b(\tau) - (R_0/R)] = r_0 + x\,(b - r_0),$$

where x varies within the limits $0 \le x \le 1$. We obtain the following modification of the one-dimensional model (8.1)-(8.4):

$$\left(c\rho - hH \frac{\partial v}{\partial u}\right) \frac{\partial u}{\partial \tau} = \frac{\lambda}{[b - (R_0/R)]^2} \frac{\partial^2 u}{\partial x^2} + \frac{\lambda}{[r_0 + x\,(b - r_0)]\,(b - r_0)} \frac{\partial u}{\partial x} +$$

$$+ \frac{1}{(b - r_0)^2} \frac{\partial \lambda}{\partial u} \left(\frac{\partial u}{\partial x}\right)^2 - \frac{2\alpha R^2}{\lambda_0 \Delta} u, \quad \tau > 0, \quad 0 \le x \le 1, \tag{11.1}$$

$$u = (f_2(t) - T_\infty)/(T_{\text{lev}} - T_\infty), \quad x = 1, \tag{11.2}$$

$$(\lambda/(b - r_0)) \frac{\partial u}{\partial x} = (c_2\rho_2/c_0\rho_0)\, r_0 \frac{\partial u}{\partial \tau}, \quad x = 0, \tag{11.3}$$

$$u = 0 \quad \text{at} \quad \tau = 0, \quad 0 \le x \le 1. \tag{11.4}$$

CONCLUSIONS

1. We have obtained mathematical models of the process of heating an abrasive disk. They generalize and refine the mathematical model suggested in [1].

2. Proceeding from a number of natural assumptions, we have obtained additional information about the process of heating:

a) the time of the process increases with increase in the endothermal effect of the physicochemical transformations (dimensionless number h), i.e., the durability of the disk grows;

b) the time of the process decreases with increase in u_r, while the rate of the process increases, i.e., the durability of the disk decreases.

3. We have revealed the possibilities for an increase in the efficiency of operation of the abrasive tool and its durability by controlling the physicochemical processes in the cutting zone.

4. Mathematical models (4.1)-(4.5), (8.1)-(8.4), (11.1)-(11.4) can be taken as a basis for calculating the heating process on a computer by some numerical method.

NOTATION

c_1, ρ_1, λ_1, heat capacity, J/(kg·K), density, kg/m^3, and thermal conductivity, W/(m·K), of the abrasive disk, respectively; c_2, ρ_2, heat capacity and density of the metal axis on which the disk is clamped; H_1, enthalpy of the physicochemical transformations, J/m^3; v, concentration of the reagent, %; α, heat transfer coefficient, W/(m^2·K); T_∞, temperature of the external medium, K; T_0, initial temperature of the disk, K; T_y, temperature developed on the working surface of the disk, K; T, temperature of the abrasive disk, K; r, z, radial and transverse coordinates; R, R_0, external and internal radii of the disk, mm; Δ, thickness of the disk, mm; t, time, sec; t_0, time of arrival of arrival of the working-surface temperature at a steady state, sec.

REFERENCES

1. P. I. Yashcheritsin, A. K. Tsokur, and M. L. Eremenko, Thermal Phenomena in Grinding and Properties of Treated Surfaces [in Russian], Minsk (1973).
2. A. K. Tsokur, A. Ya. Tsokur, and A. I. Draevskii, Inzh.-Fiz. Zh., **56**, No. 6, 1008-1013 (1989).
3. P. I. Yashcheritsin, A. K. Tsokur, and A. I. Draevskii, Vestsi Akad. Navuk BSSR, Ser. Fiz.-Tekh. Navuk, No. 2, 43-48 (1986).
4. A. V. Luikov, Theory of Heat Conduction [in Russian], Moscow (1967).